3 Probability

3.1 Introduction to Probability

When running your own business you will have to make decisions based on your analysis of uncertainties. For example, you may want to know the likelihood that sales will increase if you decrease the price of your product, or how the weather will impact on your sales. Probability is a way of representing the likelihood of a particular event occuring. To understand probability we have to understand the vocabulary used in probability.

- **Trial:** An action which results in one of several possible outcomes. For example, a trial is tossing a coin and the outcomes are head or tails. Another possible trial is rolling a die with the possible outcomes being to roll a 1, 2, 3, 4, 5 or 6.
- **Experiment:** A series of trials. For example, tossing a coin two or more times would be defined as an experiment.

Example 3.1.1

Write down all of the possible outcomes of the experiment involving tossing a coin twice.

When we toss a single coin the possible outcomes are either heads or tails $\{(H,T)\}$. If we toss a coin twice the possible outcomes of the experiment are $\{(H,H),(H,T),(T,H),(T,T)\}$.

- **Sample space**: The set of all possible outcomes of a trial or experiment. The possible outcomes are known as **sample points**.
- **Event**: A collection of experimental outcomes with some common characteristic. For example, if we roll two dice, we could define the event of rolling the same number on both dice, i.e. $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$.

3.1.1 Probability Rules

There are two main rules when dealing with the probability of an outcome. First, the probability P of an event E occuring is denoted by $P(E)$ and takes a value between 0 and 1, i.e.

$$0 \leq P(E) \leq 1$$

If an outcome E has a probability of 0 is means that the outcome is impossible, and if the event has a probability of 1, then it is a certainty.

In addition to this, the sum of all the probabilities of all possible outcomes should add up to 1, i.e.

$$P(E_1) + P(E_2) + P(E_3)\ldots P(E_i) = 1$$

If we have some event E, for example rolling an even number on a die, the **complement** of E which is denoted by E' consists of all the other outcomes in the sample space in E, i.e. rolling an odd number on a die.

$$P(E) + P(E') = 1$$

3.1.2 Methods of Calculating Probabilities

There are various different methods for calculating the probability of some event occuring.

Classical Approach

The clasical approach to calculating probability is used when we know in advance that all of the possible experimental outcomes are equally likely to occur. If there are n possible outcomes, we assign each outcome with the same probability $\frac{1}{n}$. For example, consider rolling a fair die. If a die is fair, there are six possible outcomes each with the same chance of occuring. Therefore the probability of rolling a 2 would be $P(2) = \frac{1}{6}$. The classical approach to calculated probabilities is limited to cases where there are equal chances of each event occuring.

Relative Frequency

We can use experiments to calculate the probability of some event E occuring. The experiment should be repeated a large number of times with the total number of times event E being recorded, in addition to the total number of times the experiment was conducted. The probability of event E occuring is given by:

$$P(E) = \frac{\text{Total number of times } E \text{ occured}}{\text{Total number of times the experiment was conducted}}$$

There are limitations to the relative frequency approach due to the fact that the experiment needs to be repeated a large number of times, which may be difficult, and repeating the experiment under the same conditions can also be challenging.

Example 3.1.2

An unfair die is rolled 200 times and the following results were obtained:

Outcome	Frequency
1	15
2	10
3	22
4	108
5	20
6	25

Calculate the probability of rolling a 4.

To calculate the probability of rolling a 4 we use the following:

$$P(E) = \frac{\text{Total number of times } E \text{ occured}}{\text{Total number of times the experiment was conducted}}$$

As we are looking for the probability of rolling a 4, we see that 4 occurs 108 times when the experiment is repeated 200 times.

$$P(4) = \frac{\text{Total number of times 4 occured}}{\text{Total number of times the experiment was conducted}} = \frac{108}{200} = \frac{27}{50} = 0.54$$

Subjective Approach

We use a subjective approach when we have no, or very little, information about an event occuring. We therefore assign a probability based on our own experience or intuition. For example you may make a business decision based on your experience of what has occured in the past.

3.1.3 Types of Events

If we consider two or more events they can be one of the following:

- **Mutually exclusive:** If events are mutually exclusive then they cannot occur at the same time. For example you cannot toss a coin and land on a head and tails at the same time, or if you draw a card from a pack, you cannot draw a red card and a black card.
- **Independent:** If two events are independent then the occurence of one event happening has no impact on the second event occuring. For example, the probability of rolling a 6 on a die and tossing a tail on a coin do not depend upon each other.

3.1.4 Rules of Probability

There are several laws of probability that you should recall when dealing with probability.

Addition Law

When we calculate the probability of two events occuring we use the addition law. The probability of events A or B occuring is defined as the union of A and B and is denoted as:

$$P(A \cup B) = P(A \text{ or } B)$$

The union of A and B is given by the venn diagram below: